Dérivation de fonctions composées

Formule générale : $\left(f\left(u(x)\right)\right)' = u'(x) \times f'\left(u(x)\right)$

EXERCICE 2B.1

$$\left(\sqrt{u(x)}\right)' = u'(x) \times \frac{1}{2\sqrt{u(x)}} = \frac{u'(x)}{2\sqrt{u(x)}}$$

1. $f(x) = \sqrt{9x-5}$ $u(x) =$ $u'(x) =$ $f'(x) =$	**2.** $f(x) = \sqrt{3x^2+6x-1}$ $u(x) =$ $u'(x) =$ $f'(x) =$	**3.** $f(x) = \sqrt{\dfrac{2x+1}{x-1}}$ $u(x) =$ $u'(x) =$ $f'(x) =$

EXERCICE 2B.2

$$\left(\left(u(x)\right)^n\right)' = n \times \left(u(x)\right)^{n-1} \times u'(x)$$

1. $f(x) = \left(4x^2+2x-7\right)^5$ $u(x) =$ $u'(x) =$ $f'(x) =$	**2.** $f(x) = \left(\dfrac{x+1}{x-1}\right)^2$ $u(x) =$ $u'(x) =$ $f'(x) =$
3. $f(x) = \left(\dfrac{x^3+2}{3x-5}\right)^4$ $u(x) =$ $u'(x) =$ $f'(x) =$	

EXERCICE 2B.3

Dériver la fonction f définie par $f(x) = \left(3-x^2\right)\sqrt{x^2-3x+7}$

$u(x) = \ldots\ldots\ldots\ldots$ → $u'(x) = \ldots\ldots\ldots\ldots$ et $v(x) = \sqrt{\ldots\ldots\ldots\ldots}$ → $v'(x) = \dfrac{\ldots\ldots\ldots\ldots}{2\sqrt{\ldots\ldots\ldots\ldots}}$

$$f'(x) = \ldots\ldots\sqrt{\ldots\ldots\ldots\ldots} + (\ldots\ldots\ldots) \times \frac{\ldots\ldots\ldots}{2\sqrt{x^2-3x+7}} = \frac{\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots}{2\sqrt{x^2-3x+7}}$$

$= \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

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EXERCICE 2B.1

$$\left(\sqrt{u(x)}\right)' = u'(x)\times\frac{1}{2\sqrt{u(x)}} = \frac{u'(x)}{2\sqrt{u(x)}}$$

1. $f(x)=\sqrt{9x-5}$

$u(x)=9x-5$

$u'(x)=9$

$f'(x)=\dfrac{9}{2\sqrt{9x-5}}$

2. $f(x)=\sqrt{3x^2+6x-1}$

$u(x)=3x^2+6x-1$

$u'(x)=6x+6$

$f'(x)=\dfrac{6x+6}{2\sqrt{3x^2+6x-1}}=\dfrac{3x+3}{\sqrt{3x^2+6x-1}}$

3. $f(x)=\sqrt{\dfrac{2x+1}{x-1}}$

$u(x)=\dfrac{2x+1}{x-1}$

$u'(x)=\dfrac{2(x-1)-(2x+1)}{(x-1)^2}$

$=\dfrac{2x-2-2x-2}{(x-1)^2}=\dfrac{-4}{(x-1)^2}$

Donc $f'(x)=\dfrac{-4}{(x-1)^2}\times\dfrac{1}{2\sqrt{\dfrac{2x+1}{x-1}}}$

$=\dfrac{-2}{(x-1)^2}\times\sqrt{\dfrac{x-1}{2x+1}}$

EXERCICE 2B.2

$$\left((u(x))^n\right)' = n\times(u(x))^{n-1}\times u'(x)$$

1. $f(x)=(4x^2+2x-7)^5$

$u(x)=4x^2+2x-7$

$u'(x)=8x+2$

$f'(x)=5(4x^2+2x-7)^4(8x+2)$

2. $f(x)=\left(\dfrac{x+1}{x-1}\right)^2$

$u(x)=\dfrac{x+1}{x-1}$ $\rightarrow u'(x)=\dfrac{(x-1)-(x+1)}{(x-1)^2}=\dfrac{-2}{(x-1)^2}$

$f'(x)=2\left(\dfrac{x+1}{x-1}\right)\times\dfrac{-2}{(x-1)^2}=\dfrac{-4(x+1)}{(x-1)^3}$

3. $f(x)=\left(\dfrac{x^3+2}{3x-5}\right)^4$

$u(x)=\dfrac{x^3+2}{3x-5}$ $\rightarrow u'(x)=\dfrac{3x^2(3x-5)-(x^3+2)\times 3}{(3x-5)^2}=\dfrac{9x^3-15x^2-3x^3-6}{(3x-5)^2}=\dfrac{6x^3-15x^2-6}{(3x-5)^2}$

$f'(x)=4\left(\dfrac{x^3+2}{3x-5}\right)^3\times\dfrac{6x^3-15x^2-6}{(3x-5)^2}$

EXERCICE 2B.3

Dériver la fonction f définie par $f(x)=(3-x^2)\sqrt{x^2-3x+7}$

$u(x)=3-x^2$ $\rightarrow u'(x)=-2x$ et $v(x)=\sqrt{x^2-3x+7}$ $\rightarrow v'(x)=\dfrac{2x-3}{2\sqrt{x^2-3x+7}}$

$f'(x)=(-2x)\sqrt{x^2-3x+7}+(3-x^2)\times\dfrac{2x-3}{2\sqrt{x^2-3x+7}}=\dfrac{(-2x)\times 2(x^2-3x+7)+(3-x^2)(2x-3)}{2\sqrt{x^2-3x+7}}$

$=\dfrac{-4x^3+12x^2-28x+6x-9-2x^3+3x^2}{2\sqrt{x^2-3x+7}}=\dfrac{-6x^3+15x^2-22x-9}{2\sqrt{x^2-3x+7}}$