EXERCICES 3B.1 Déterminer une primitive d'une fonction f de la forme : $\frac{U'}{U^n} = U'(x) \times U^{-n}(x)$

a. $f(x) = \frac{9x^2}{(3x^3+5)^2} = \ldots\ldots$ →on pose $u(x) = \ldots\ldots$ donc $u'(x) = \ldots\ldots$

→ainsi : $f(x) = \ldots\ldots$ et $F(x) = \frac{\ldots\ldots}{\ldots} = \frac{\ldots\ldots}{\ldots}$

b. $f(x) = \frac{2x-2}{(7-2x+x^2)^3} = \ldots\ldots$ → on pose $u(x) = \ldots\ldots$, $u'(x) = \ldots\ldots$

→ainsi : $f(x) = \ldots\ldots$ et $F(x) = \frac{\ldots\ldots}{\ldots} = \frac{\ldots\ldots}{\ldots} = \frac{\ldots\ldots}{\ldots\ldots}$

c. $f(x) = \frac{14x-14}{(7x^2-14x)^4} = \ldots\ldots$ → on pose $u(x) = \ldots\ldots$ donc $u'(x) = \ldots\ldots$

→ainsi : $f(x) = \ldots\ldots$ et $F(x) = \frac{\ldots\ldots}{\ldots} = \frac{\ldots\ldots}{\ldots} = \frac{\ldots\ldots}{\ldots\ldots}$

d. $f(x) = \frac{2x-3}{(x^2-3x+5)^2} = \ldots\ldots$ →on pose $u(x) = \ldots\ldots$, $u'(x) = \ldots\ldots$

→ainsi : $f(x) = \ldots\ldots$ et $F(x) = \frac{\ldots\ldots}{\ldots} = \frac{\ldots\ldots}{\ldots} = \frac{\ldots\ldots}{\ldots\ldots}$

EXERCICES 3B.2

Transformer f pour faire apparaître une forme $k \times \frac{U'}{U^n} = k \times U'(x) \times U^{-n}(x)$ puis déterminer une primitive

a. $f(x) = \frac{x-2}{(x^2-4x)^2} = \ldots\ldots$ →on pose $u(x) = \ldots\ldots$, $u'(x) = \ldots\ldots$

→ainsi : $f(x) = \ldots\ldots$ et $F(x) = \ldots\ldots\frac{\ldots\ldots}{\ldots} = \frac{\ldots\ldots}{\ldots\ldots}\ldots\ldots$

b. $f(x) = \frac{10}{(5x+9)^3} = \ldots\ldots$ → on pose $u(x) = \ldots\ldots$, $u'(x) = \ldots\ldots$

→ainsi : $f(x) = \ldots\ldots$ et $F(x) = \ldots\ldots\frac{\ldots\ldots}{\ldots} = \frac{\ldots\ldots}{\ldots\ldots}\ldots\ldots$

c. $f(x) = \frac{6x+12}{(x^2+4x+8)^4} = \ldots\ldots$ → on pose $u(x) = \ldots\ldots$, $u'(x) = \ldots\ldots$

→ainsi : $f(x) = \ldots\ldots$ et $F(x) = \ldots\ldots\frac{\ldots\ldots}{\ldots} = \frac{\ldots\ldots}{\ldots\ldots}\ldots\ldots$

d. $f(x) = \frac{6x^2-14}{(x^3-7x)^5} = \ldots\ldots$ →on pose $u(x) = \ldots\ldots$, $u'(x) = \ldots\ldots$

→ainsi : $f(x) = \ldots\ldots$ et $F(x) = \ldots\ldots\frac{\ldots\ldots}{\ldots} = \frac{\ldots\ldots}{\ldots\ldots}\ldots\ldots$

CORRIGE – Notre Dame de La Merci – Montpellier – M. Quet

EXERCICES 3B.1 *Déterminer une primitive d'une fonction f de la forme :* $\dfrac{U'}{U^n}=U'(x)\times U^{-n}(x)$

a. $f(x)=\dfrac{9x^2}{\left(3x^3+5\right)^2}=9x^2\left(3x^3+5\right)^{-2}$ →on pose $u(x)=3x^3+5$ donc $u'(x)=9x^2$

→ainsi : $f(x)=\dfrac{u'(x)}{\left(u(x)\right)^2}$ et $F(x)=\dfrac{-1}{u(x)}=\dfrac{-1}{3x^3+5}$

b. $f(x)=\dfrac{2x-2}{\left(7-2x+x^2\right)^3}=(2x-2)\left(7-2x+x^2\right)^{-3}$ → on pose $u(x)=7-2x+x^2$, $u'(x)=-2+2x$

→ainsi : $f(x)=u'(x)\times\left(u(x)\right)^{-3}$ et $F(x)=\dfrac{\left(u(x)\right)^{-3+1}}{-3+1}=\dfrac{\left(u(x)\right)^{-2}}{-2}=\dfrac{-1}{2\left(7-2x+x^2\right)^2}$

c. $f(x)=\dfrac{14x-14}{\left(7x^2-14x\right)^4}=(14x-14)\left(7x^2-14x\right)^{-4}$ → on pose $u(x)=7x^2-14x$ donc $u'(x)=14x-14$

→ainsi : $f(x)=u'(x)\times\left(u(x)\right)^{-4}$ et $F(x)=\dfrac{\left(u(x)\right)^{-4+1}}{-4+1}=\dfrac{\left(u(x)\right)^{-3}}{-3}=\dfrac{-1}{3\left(7x^2-14x\right)^3}$

d. $f(x)=\dfrac{2x-3}{\left(x^2-3x+5\right)^2}=(2x-3)\left(x^2-3x+5\right)^{-2}$ →on pose $u(x)=x^2-3x+5$ donc $u'(x)=2x-3$

→ $f(x)=u'(x)\times\left(u(x)\right)^{-2}$ et $F(x)=\dfrac{\left(u(x)\right)^{-2+1}}{-2+1}=-\left(u(x)\right)^{-1}=\dfrac{-1}{u(x)}=\dfrac{-1}{x^2-3x+5}$

EXERCICES 3B.2

a. $f(x)=\dfrac{x-2}{\left(x^2-4x\right)^2}=(x-2)\left(x^2-4x\right)^{-2}$ →on pose $u(x)=x^2-4x$ donc $u'(x)=2x-4=2(x-2)$

→ainsi : $f(x)=\dfrac{1}{2}(2x-4)\left(x^2-4x\right)^{-2}=\dfrac{1}{2}\times u'(x)\times\left(u(x)\right)^{-2}$

et $F(x)=\dfrac{1}{2}\times\dfrac{\left(u(x)\right)^{-2+1}}{-2+1}=\dfrac{\left(u(x)\right)^{-1}}{-2}=\dfrac{-1}{2u(x)}=\dfrac{-1}{2\left(x^2-4x\right)}$

b. $f(x)=\dfrac{10}{(5x+9)^3}=10(5x+9)^{-3}$ → on pose $u(x)=5x+9$ donc $u'(x)=5$

→ainsi : $f(x)=2\times5(5x+9)^{-3}=2\times u'(x)\times\left(u(x)\right)^{-3}$

et $F(x)=2\times\dfrac{\left(u(x)\right)^{-3+1}}{-3+1}=2\times\dfrac{\left(u(x)\right)^{-2}}{-2}=\dfrac{-1}{\left(u(x)\right)^2}=\dfrac{-1}{(5x+9)^2}$

c. $f(x)=\dfrac{6x+12}{\left(x^2+4x+8\right)^4}=(6x+12)\left(x^2+4x+8\right)^{-4}$ → on pose $u(x)=x^2+4x+8$ donc $u'(x)=2x+4$

→ainsi : $f(x)=3(2x+4)\left(x^2+4x+8\right)^{-4}=3\times u'(x)\times\left(u(x)\right)^{-4}$

et $F(x)=3\times\dfrac{\left(u(x)\right)^{-4+1}}{-4+1}=3\times\dfrac{\left(u(x)\right)^{-3}}{-3}=\dfrac{-1}{\left(u(x)\right)^3}=\dfrac{-1}{\left(x^2+4x+8\right)^3}$

d. $f(x)=\dfrac{6x^2-14}{\left(x^3-7x\right)^5}=\left(6x^2-14\right)\left(x^3-7x\right)^{-5}$ →on pose $u(x)=x^3-7x$ donc $u'(x)=3x^2-7$

→ainsi : $f(x)=2\left(3x^2-7\right)\left(x^3-7x\right)^{-5}=2\times u'(x)\times\left(u(x)\right)^{-5}$

et $F(x)=2\times\dfrac{\left(u(x)\right)^{-5+1}}{-5+1}=2\times\dfrac{\left(u(x)\right)^{-4}}{-4}=\dfrac{-1}{2\left(u(x)\right)^4}=\dfrac{-1}{2\left(x^3-7x\right)^4}$